REPORT DOCUMENTATION PAGE

1

Form Approved OMB NO. 0704-0188

1. REPORT DATE (DD-MM-YYYY)	2. REPORT TYPE	3. DATES COVERED (From - To)
05-09-2014	Conference Proceeding	-

4. TITLE AND SUBTITLE	
ML-PMHT Track Detection Threshold Determination for K-Distributed Clutter	5a. CONTRACT NUMBER W911NF-10-1-0369
	5b. GRANT NUMBER
	5c. PROGRAM ELEMENT NUMBER 611102
6. AUTHORS S. Schoenecker, P. Willett, Y. Bar-Shalom	5d. PROJECT NUMBER
	5e. TASK NUMBER
	5f. WORK UNIT NUMBER

7. PERFORMING ORGANIZATION NAMES AND ADDRESSES	8. PERFORMING ORGANIZATION REPORT NUMBER
University of Connecticut - Storrs 438 Whitney Road Ext., Unit 1133 Storrs, CT 06269 -1133	
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS (ES) U.S. Army Research Office P.O. Box 12211 Research Triangle Park, NC 27709-2211	10. SPONSOR/MONITOR'S ACRONYM(S) ARO 11. SPONSOR/MONITOR'S REPORT NUMBER(S) 57823-CS.86

12. DISTRIBUTION AVAILIBILITY STATEMENT

Approved for public release; distribution is unlimited.

13. SUPPLEMENTARY NOTES

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15. SUBJECT TERMS

ML-PMHT,Multistatic, Bistatic, Active Sonar, Tracking, Threshold Determination, K-distribution

16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT	15. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON
a. REPORT	b. ABSTRACT	c. THIS PAGE			Yaakov Bar-Shalom
UU	UU	UU	UU		19b. TELEPHONE NUMBER 860-486-4823

Standard Form 298 (Rev 8/98)
Prescribed by ANSI Std. Z39.18

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Conference Name: SPIE Conf. Signal and Data Processing of Small Targets, #9092-23, Baltimore,MD, May 2014.

Conference Date: May 15, 2014

ML-PMHT Track Detection Threshold Determination for K-Distributed Clutter

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Recent work developed a novel method for determining tracking thresholds for the Maximum Likelihood Probabilistic Multi-Hypothesis Tracker (ML-PMHT). Under certain "ideal" conditions, probability density functions (PDFs) for the peak points in the ML-PMHT log-likelihood ratio (LLR) due to just clutter measurements could be calculated. Analysis of these clutter-induced peak PDFs allowed for the calculation of tracking thresholds, which previously had to be done with time-consuming Monte Carlo simulations. However, this work was done for a very specific case: the amplitudes of both target and clutter measurements followed Rayleigh distributions. The Rayleigh distribution is a very light-tailed distribution, and it can be overly optimistic in predicting that high-SNR measurements are target-originated. This work examines the case where the clutter amplitudes do not follow a Rayleigh distribution at all, but instead follow a K-distribution, which more accurately describes active acoustic clutter. This will provide a framework for determining accurate tracking thresholds for the ML-PMHT algorithm.

Keywords: ML-PMHT, Multistatic, Bistatic, Active Sonar, Tracking, Threshold Determination, K-distribution

1. INTRODUCTION AND ML-PMHT BACKGROUND

The Maximum Likelihood Probabilistic Multi-Hypothesis Tracker (ML-PMHT) is a powerful tracking algorithm that can be implemented in a multistatic active sonar framework. It has its origin in a similar algorithm, the Maximum Likelihood Probabilistic Data Association (ML-PDA) tracker, which was first developed in [16] and was expanded in [17] and [9]. The ML-PMHT algorithm followed from these, as well as the work in [5], [23], [24], and [25]. The development of the ML-PMHT algorithm as a multistatic sonar algorithm is described in detail in [22] — it is only briefly summarized here.

At its heart, ML-PMHT is a very simple algorithm. Several assumptions about the target and the environment are made [6], and using these assumptions, a log-likelihood ratio is constructed. This log-likelihood ratio for a single target is given by

$$\Lambda(\mathbf{x}, Z) = \sum_{i=1}^{N_w} \sum_{j=1}^{m_i} \ln \left\{ 1 + \frac{\pi_1}{\pi_0} V \rho_j(i) p[\mathbf{z}_j(i)|\mathbf{x}] \right\} \tag{1}$$

where $\mathbf{x}$ is a target parameter vector, Z is the entire set of measurements in a batch of scans, N_w is the number of scans in the batch, m_i is the number of measurements in the i^{th} scan, π_1 is the prior probability that any given measurement is from the target, π_0 is the prior probability that any given measurement is from clutter,

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Proc. SPIE Conf. Signal and Data Processing of Small Targets, #9092-23, Baltimore, MD, May 2014.
Peter Willett was supported by the Office of Naval Research under contract N00014-13-1-0231.
Yaakov Bar-Shalom was supported by the Army Research Office under contract W911NF-10-1-0369 and the Office of Naval Research under contract N00014-10-1-0029.

V is the measurement search volume, $\rho_j(i)$ is the amplitude likelihood ratio for the j^{th} measurement in the i^{th} scan, $\mathbf{z}_j(i)$ is the j^{th} measurement in the i^{th} scan, and $p[\mathbf{z}_j(i)|\mathbf{x}]$ is a target-centered Gaussian. It is stressed that ML-PMHT is a batch tracker, as opposed to a recursive tracker such a Kalman filter, and it is non-Bayesian — it assumes that the target follows some deterministic (unknown) parameterizable trajectory which is corrupted by measurement noise.

To apply ML-PMHT as a multistatic tracker, a batch of measurements is accumulated and inserted into (1), and the LLR is optimized over the target parameter vector $\mathbf{x}$. If the maximum LLR value found is above a certain threshold, a target is declared; if the maximum LLR value is below this threshold, a no-target condition is assumed. A problematic part of actually implementing ML-PMHT is determining exactly what this threshold should be. In some ML-PDA/ML-PMHT implementations [14], [19], and [20], "optimal" thresholds were determined mainly through trial and error (which obviously predicates use of the tracker in real-time). Work in [7] did develop a method for determining thresholds empirically via Monte Carlo testing, but this approach is very computationally intensive and time consuming; again, it is not possible to use this method of threshold determination with real-time processing. A fast and accurate method for ML-PMHT threshold determination is required.

2. DETERMINING ML-PMHT TRACKING THRESHOLDS

Recent work in [21] developed a method for determining ML-PMHT tracking thresholds theoretically (and rapidly), but it was limited to only several specific types of clutter measurements. This work is summarized below and is then expanded to a more general framework that numerically computes ML-PMHT tracking thresholds for a wider variety of cases. The resultant overall framework for determining clutter thresholds is shown in Figure 1.

2.1 Determining peak ML-PMHT LLR PDFs

The clutter threshold determination starts with an individual clutter measurement. In this work, the focus is on 3-dimensional measurements. The individual measurements are azimuth, time-delay, and range rate. Consider (1) for just a single measurement This can be written as

$$\lambda_1(\mathbf{x}, \mathbf{z}) = \ln \left\{ 1 + K e^{-\left[\frac{(z_1-\mu_1)^2}{2\sigma_1^2} + \frac{(z_2-\mu_2)^2}{2\sigma_2^2} + \frac{(z_3-\mu_3)^2}{2\sigma_3^2} \right]} \right\} \tag{2}$$

where

$$K = \frac{\pi_1}{\pi_0} \frac{V_1 V_2 V_3}{\sqrt{8\pi^3}\sigma_1\sigma_2\sigma_3} \tag{3}$$

Here, μ_1, μ_2 and μ_3 are the mean target locations in measurement space, σ_1, σ_2 and σ_3 are the respective measurement errors, and V_1 , V_2, and V_3 are the individual-dimension measurement search volumes. For the time being, it is assumed that no amplitude information is being processed, so $\rho_j(i) = 1$.

One of the assumptions of the ML-PMHT algorithm is that clutter measurements are uniformly distributed in measurement space; with this, (2) can be treated as simply a random variable (RV) transformation; i.e. if Z_1, Z_2, and Z_3 are uniformly distributed RVs in azimuthal, time-delay, and range-rate measurement space, then W is simply a new transformed RV, where

$$W \triangleq \lambda_1(\mathbf{x}, \mathbf{z}) \tag{4}$$

Using standard random variable transformation techniques, then, it is possible to obtain the PDF for W under various cases. This "single-measurement PDF" of W represents the range of values (with their corresponding probability densities) that the ML-PMHT LLR can take on from a single clutter measurement. In [21], analytic expressions for the single-measurement PDFs were derived for the following cases: 1-dimensional measurements (time-delay only or azimuth only), 2-dimensional measurements (azimuth and time-delay), 3-dimensional measurements (azimuth, time-delay, and range-rate), and 2-dimensional measurements with Rayleigh-distributed amplitudes for both target- and clutter-originated measurements.

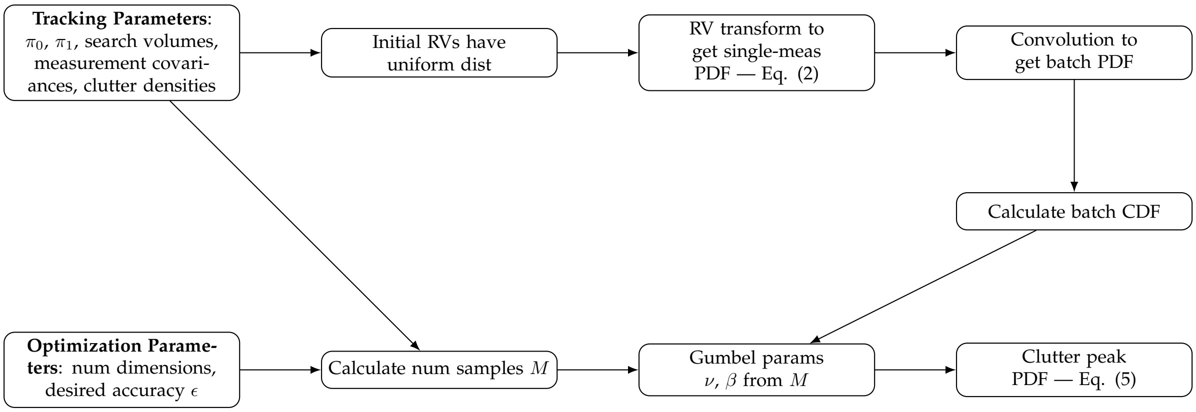

Figure 1. Framework for determining clutter peak PDF

ML-PMHT is a batch tracker; it is necessary to obtain the PDF of a batch of measurements, instead of just the PDF of a single measurement. Fortunately, another one of the ML-PMHT assumptions (and this can be seen in the structure of the ML-PMHT LLR) is that measurements at different times are assumed to be independent. Thus, the single-measurement RVs resulting from the transformation described by (4) will also be independent (and identically distributed). As a result, the PDF of a batch of N measurements can be computed simply by convolving the single-measurement PDF of W $N-1$ times (the PDF of a sum of two independent RVs is just the convolution of the individual RV PDFs). This "batch PDF" probabilistically describes the range of possible values of the ML-PMHT LLR (1).

This batch PDF describes *all* possible values for the ML-PMHT LLR; we are concerned with the *peak* value of the ML-PMHT LLR. To obtain the PDF for this peak value, extreme-value (EV) theory is used [8], [10], [11], [12], [13], [15]. According to EV theory, in this case, the PDF of the peak value of the LLR can be approximated with a Gumbel distribution, which is expressed as

$$f(x) = \frac{1}{\beta} \exp\left[-\left(\frac{x-\nu}{\beta} \right) - \exp\left(-\frac{x-\nu}{\beta} \right) \right] \tag{5}$$

where ν is the location parameter and β is the shape parameter. In an "actual" ML-PMHT implementation, a non-linear numerical optimization scheme would be run to optimize (1) over $\mathbf{x}$. In theory, if the batch ML-PMHT LLR is sampled M times, where M is some large number, the value of the maximum sample should be the same as the maximum LLR value determined by numerical optimization (i.e. both results would come equally close to the "true" LLR maximum). With that idea in mind, if the accuracy of the actual optimizer is known, then is possible to compute a value for M that would produce (via sampling) a maximum with this same accuracy. From extreme-value theory, the PDF of the maximum sample from a series of length M IID RV samples takes on a Gumbel distribution. The parameters ν and β in (5) are a function of M and the underlying cumulative distribution function (CDF) of the sampled RV. The key point is that the numerical optimization is represented as taking the maximum from a large sample of IID RVs; we compute how many times it is necessary to sample the batch PDF to obtain the same accuracy as the numerical optimization achieves (note that actual optimization via sampling is *not* done), and then this value M is used along with the CDF of the underlying batch RV to determine the "peak" PDF — this statistically characterizes the peak point in the ML-PMHT LLR. Finally, the Neyman-Pearson Lemma from classical detection theory [18] is used — a desired probability of false track (P_{FT}) is selected, which in turn determines the tracking threshold.

One of the steps in this process is to perform the transformation of a uniformly distributed RV to the single-measurement RV using the single-measurement ML-PMHT LLR as the transforming function. Again, this was done analytically in [21] for four cases: 1-dimensional measurements, 2-dimensional measurements, 3-dimensional measurements, and 2-dimensional measurements with a Rayleigh amplitude ratio. It does not appear to be possible to analytically calculate the expression for a single-measurement transformed RV for the

2-dimensional case with a clutter amplitude distribution other than Rayleigh, nor does it appear to be possible to calculate the transformed PDF for *any* amplitude distribution in the 3-dimensional case (the integrals for the probability transformations are intractable analytically).[1] In this work, numerical methods are used to compute the single-measurement PDFs for the cases where analytic results are not possible. We are particularly interested in cases where the clutter is modeled as being K-distributed, which is a more accurate model of "real" clutter [3].

2.2 Amplitude likelihood ratios

Previous work dealing with amplitudes [17] assumed that both the clutter and the target amplitude followed a Rayleigh distribution. In intensity space (amplitude squared) these become exponential distributions. (From here on, work is done in intensity space because the exponential distribution is very simple to work with.) Using this model, clutter intensity is modeled as a unit mean exponential, and the target intensity is modeled as an exponential with mean σ^2. Additionally, both the target and clutter distributions are thresholded at certain level τ, which is the sonar detector threshold. The amplitude LR that results is

$$\rho_j(i) = f(v) = \frac{1}{\sigma^2} \frac{e^{\frac{\tau - v}{\sigma^2}}}{e^{\tau - v}} \qquad v \geq \tau \tag{6}$$

where v is the intensity, σ^2 is the expected intensity, and τ is the detector threshold, in units of intensity. (It is important to note that regardless of whether measurements are treated in amplitude space or intensity space, the likelihood ratios come out the same.)

A "problem" with the Rayleigh/exponential distributions when they are used to describe clutter is that they are very "light-tailed" distributions. The result of this light tail is that the amplitude LLR for high-SNR measurements becomes very large — this is shown in Figure 3. When a large-amplitude measurement is plugged into $\rho_j(i)$ in (1), it essentially dominates all other measurements. As a result, the ML-PMHT LLR gets all of its value (and the resulting discovered target gets its positional measurement) from this single measurement. This is bad enough if the measurement is actually from the target (it is difficult to get an accurate target solution off of one measurement); it is even worse if the high-amplitude measurement is actually from clutter. In such a case, a false track is practically guaranteed.

Recent work [1], [2], and [3] has shown that the K-distribution provides a more accurate model of active acoustic clutter. The probability distribution for the clutter intensity in this case is given by

$$f(v) = \frac{2}{\lambda \Gamma(\alpha_k)} \left(\frac{v}{\lambda_k} \right)^{(\alpha_k - 1)/2} K_{\alpha_k - 1} \left(2 \sqrt{\frac{v}{\lambda_k}} \right) \qquad v > 0 \tag{7}$$

Here, α_k is a K-distribution parameter, $\lambda_k = 1/\alpha_k$, and K_ν is the Basset function (a modified Bessel function of the second kind) [4]. The parameter α_k for this distribution determines the "heaviness" of the tail; the smaller the value of α_k, the heavier the tail (i.e. the more the probability mass is spread out to the right). This is shown in Figure 2 — for large values of α_k, the distribution approaches that of the unit exponential; while for small values of α_k, the tails of the distributions become much heaver.

Such a distribution has a very interesting effect on the amplitude/intensity LLR (it is important to keep in mind for the following discussion that the *target* amplitude/intensity is still modeled as a Rayleigh/exponential RV with parameter σ^2). The amplitude LLR using K-distributed clutter is shown in Figure 4. For small SNR values, the LLR is negative, which indicates that the measurement is more likely to have been clutter-originated. For medium SNR values (10-23 dB in this case), the LLR is positive, indicating that the measurement is more likely to have been target-originated. However, for large SNR measurements, the LLR is again negative, which says that the measurement is most likely clutter-originated. This stands in stark contrast to the Rayleigh model, which mathematically says that large SNR measurements come from the target, essentially without a doubt.

[1] It was not *proven* that these cases do not have analytic solutions. However, the authors were unable after much effort to analytically solve the integrals necessary for the probability transformations.

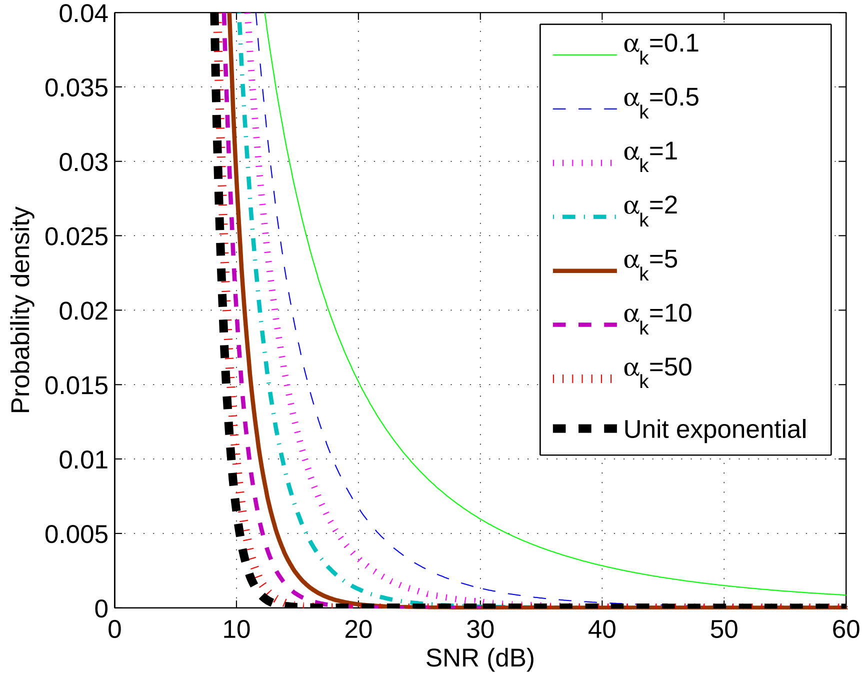

Figure 2. Squared K-distribution PDF and unit exponential PDF

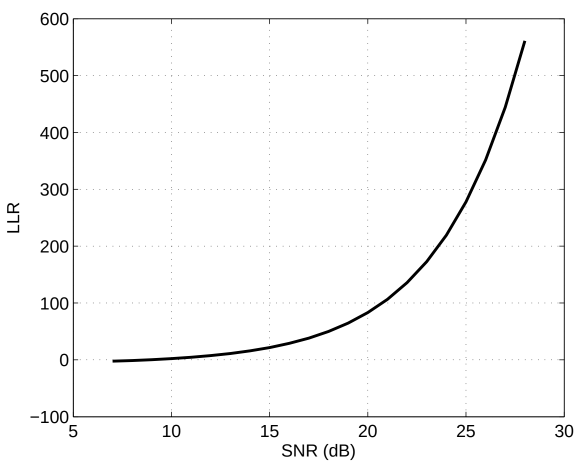

Figure 3. LLR for Rayleigh distribution clutter

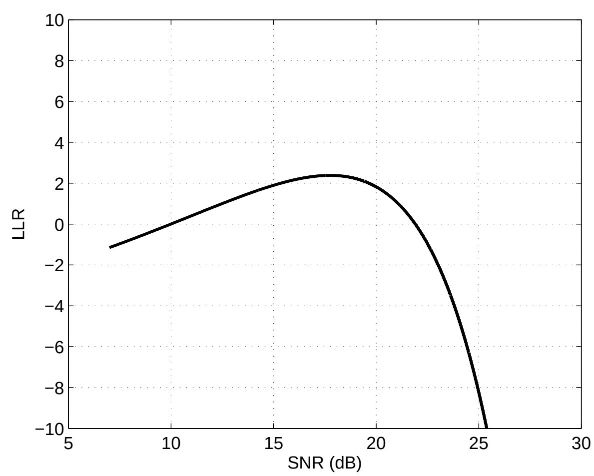

Figure 4. LLR for K-distribution clutter

2.3 Numerically computing single-measurement LLRs

At this point, it is necessary to develop a framework for numerically computing the single-measurement PDFs that result from transforming a uniform random variable by the single-measurement ML-PMHT LLR (2). This is done by rewriting the single-measurement ML-PMHT LLR in a "random variable" form

$$W = \ln(1 + Z) \tag{8}$$

where

$$Z = XY \tag{9}$$

Here W, Z, X, and Y are all random variables; we are ultimately trying to compute the PDF of W. Writing the expression for Z gives

$$Z \triangleq \underbrace{\rho_j(i)}_{X} \; \underbrace{Ke^{-\left[\frac{(z_1-\mu_1)^2}{2\sigma_1^2}+\frac{(z_2-\mu_2)^2}{2\sigma_2^2}+\frac{(z_3-\mu_3)^2}{2\sigma_3^2}\right]}}_{Y} \tag{10}$$

It is actually possible to get an analytic solution for the PDF of Y using the standard one-to-one RV transformation technique. Start by defining the exponential term as single, transformed random variable — a sum of three shifted, scaled, and squared uniform random variables

$$U = \frac{(z_1-\mu_1)^2}{2\sigma_1^2} + \frac{(z_2-\mu_2)^2}{2\sigma_2^2} + \frac{(z_3-\mu_3)^2}{2\sigma_3^2} \tag{11}$$

The PDF for U comes out to be[2]

$$p_U(u) = 2\pi\frac{\sigma_1\sigma_2\sigma_3}{V_1V_2V_3}\sqrt{u} \qquad\qquad 0 \le u \le \min\left\{ \left(\frac{\mu_1}{\sigma_1}\right)^2, \left(\frac{\mu_2}{\sigma_2}\right)^2, \left(\frac{\mu_3}{\sigma_3}\right)^2 \right\} \tag{12}$$

This in turn leads to an expression for the PDF of Y

$$p_Y(y) = 2\pi\frac{\sigma_1\sigma_2\sigma_3}{V_1V_2V_3}\left(2\ln\frac{K}{y}\right)^{\frac{1}{2}}\frac{2}{y} \qquad\qquad 0 < y \le K \tag{13}$$

The PDF for X cannot be determined analytically. To calculate it numerically, start with a squared K-distributed random variable with parameter α_k, and transform it with the amplitude/intensity LLR

$$f(v) = \frac{\frac{1}{\sigma^2}\exp[(\tau - v)/\sigma^2]}{C\frac{2}{\lambda\Gamma(\alpha_k)}\left(\frac{v}{\lambda_k}\right)^{(\alpha_k-1)/2} K_{\alpha_k-1}\left(2\sqrt{\frac{v}{\lambda_k}}\right)} \tag{14}$$

Here, C is a numerically-calculated normalization constant for a thresholded squared K-distribution PDF. Again, the transformation is performed with the standard one-to-one RV transformation method. Results for the PDF of X are shown in Figures 5, 6, and 7 for three different values of α_k. (Because some of the theoretical PDFs for X looked very "strange" — see Figure 5 as an example — all of these PDFs were verified correct with empirical testing.)

With the PDFs of X and Y now available, the PDF of $Z = XY$ can be computed by introducing the auxiliary random variable $U = Z$, performing a standard two-to-two RV transformation to get the bivariate PDF $P_{Z,U}(z, u)$, and then integrating out U to get $P_Z(z)$. Finally, the PDF of W is obtained with one final one-to-one numerical transform described by (4). Resultant PDFs for W for three values of α_k are shown together in Figure 8. (Again, note that these and all following results are for 3-dimensional measurements.)

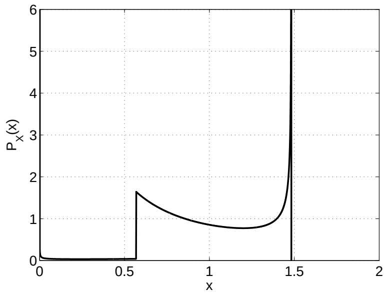

Figure 5. PDF of X for $\alpha_k = 0.1$

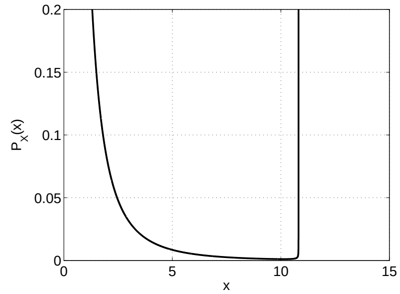

Figure 6. PDF of X for $\alpha_k = 0.5$

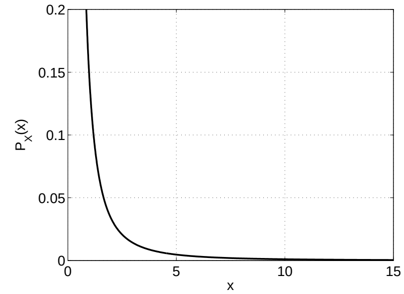

Figure 7. PDF of X for $\alpha_k = 5$

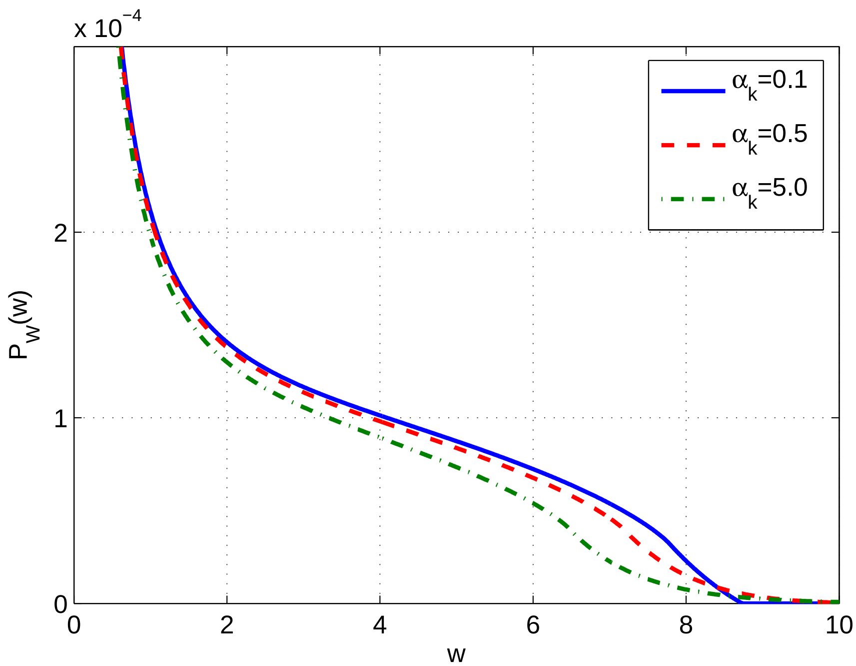

Figure 8. Single-measurement W PDFs for $\alpha_k = \{0.1, 0.5, 5.0\}$

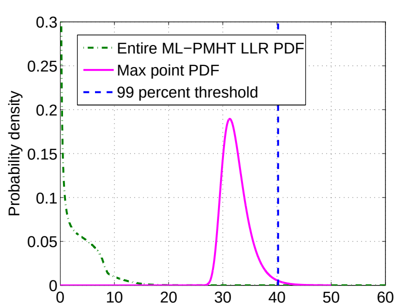

Figure 9. Batch PDFs, peak PDFs, and clutter thresholds for $\alpha = 0.1$

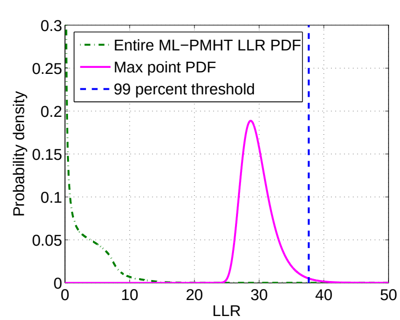

Figure 10. Batch PDFs, peak PDFs, and clutter thresholds for $\alpha = 0.5$

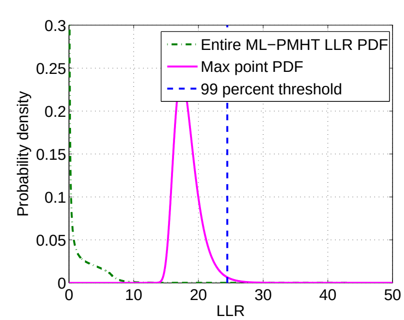

Figure 11. Batch PDFs, peak PDFs, and clutter thresholds for $\alpha = 5$

Table 1. Comparison of model vs. empirical tracking thresholds (dimensionless units for LLR). Model thresholds are from the method developed in this work, while empirical thresholds and confidence intervals are from Monte Carlo testing.

α_k	Model	Empirical	Empirical 95% Confidence Interval
0.1	40.2	39.5	[38.7, 40.3]
0.5	38.1	37.5	[37.0, 38.1]
0.8	32.2	32.9	[32.4, 33.4]
1.0	30.7	31.2	[30.5, 32.1]
2.0	28.2	28.2	[27.5, 29.0]
5.0	24.5	24.3	[23.7, 24.8]
10	22.1	22.8	[22.2, 23.3]
50	19.0	20.5	[20.1, 20.8]
Rayleigh	18.5	20.0	[19.8, 20.3]

3. RESULTS

With the single-measurement PDFs calculated numerically, the remainder of the framework shown in Figure 1 is applied. Resultant batch PDFs, extreme-value (peak) PDFs and thresholds (for $P_{FT} = 0.01$) are shown in Figures 9, 10, and 11. All of the calculated model thresholds are then compared to empirically determined thresholds, and in almost all cases the results agree extremely well with the empirical values — in most cases the model values fall within the 95 percent empirical error bounds, and for instances where this is not the case the values still are reasonably close. Also note that the results are consistent with what is expected — as α_k gets larger, the tail of the K-distribution gets lighter and lighter, approaching that of a Rayleigh. This exactly what is seen in Table 1. The results for higher values of α_k approach those of Rayleigh-distributed clutter.

The empirical values in Table 1 are "correct." They are the result of many clutter-only simulations that are run through the ML-PMHT tracker and then optimized in a Monte-Carlo fashion, but they are also computationally intensive to calculate. Each of the empirical thresholds in Table 1 took several hours of run-time to calculate. In contrast, the model values each take only several seconds to compute, and they are accurate enough to use in the ML-PMHT tracking framework.

4. CONCLUSION

We have expanded on the methods developed in [21] to develop ML-PMHT thresholds for K-distributed clutter, which is a more realistic distribution for describing active clutter amplitude/intensity measurements. Transformed single-measurement PDFs were numerically calculated for various values of α_k, the parameter that controls the K-distribution shape. These numerical PDFs were then processed through the previously developed framework to obtain batch PDFs, peak PDFs and finally ML-PMHT clutter thresholds. The ability to rapidly and accurately determine these clutter thresholds for a more realistic clutter amplitude parameterization will improve ML-PMHT as a multistatic active tracker.

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[2] This transformation is not entirely straightforward — see [21] for a detailed derivation.

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